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Heat transfer in percolated fixed beds with temperature-dependent physical properties—compressive and dispersive effects

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Abstract—This study concerns the temperature waves generated in a packed bed by a change in the inlet temperature of a percolating fluid. The shape of these waves, and their deformation as they propagate, can be considerably affected by the variation of specific heats and specific volumes with temperature. Two qualitatively different behaviours are theoretically illustrated. For example when the temperature of air, flowing through a bed of glass beads, is raised from 20 to 100°C, the resulting temperature wave is 'dispersive' in the sense that it spreads more and more as it propagates, under the effect of the changes in densities and heat capacities of the two phases. A similar but weaker effect occurs for systems CO_2–glass and water–glass. On the other hand, a bed of lead beads percolated by carbon dioxide will exhibit a 'compressive' trend resulting in a steady and sharp 'shock wave'. All trends are reversed when a cooling step is considered instead of a heating step. These effects are illustrated using a simplified model that neglects convective dispersion and heat transfer resistances between the two phases. The quantitative importance of the spreading is compared to that due to convective axial dispersion, by evaluating the corresponding Péclet numbers. Depending on the operating conditions the effect illustrated may be predominant (mostly in gas–solid systems and/or at low Reynolds number) or negligible (mostly in liquid–solid systems and/or at high Reynolds). Accounting for the spreading effect (or sharpening effect) due to property variations is thought to be important in the interpretation of fixed-bed experiments, for example when heat transfer coefficients are evaluated from such experiments.

INTRODUCTION

NUMEROUS models have been developed for describing the propagation of temperature fronts in packed beds percolated by a fluid [1–10]. These models take into account various non-idealities such as flow dispersion and diffusional heat transfer in the fluid phase and in the solid phase. By contrast, the physical properties of the fluid and the solid (densities, specific heats) are usually treated as averaged constants. The aim of the present text is not to propose an additional model, but to give some insight into the effect of the variability of these physical properties with temperature, in the range 0–100°C. For this purpose, we shall neglect all other non-idealities, and assume non-dispersed piston flow, local thermal equilibrium between phases and negligible pressure gradients in the bed. On the other hand, we shall consider that the enthalpies of the phases are non-linear functions of temperature through variable specific heats and variable specific volumes of the fluid and the solid. The effect of these assumptions on the shape (the dispersion) of the temperature fronts will then be discussed.

CONSERVATION EQUATIONS AND GENERAL SOLUTIONS

With the assumptions mentioned above, we may write a differential enthalpy balance, a differential mass conservation relation for the fluid and a mass conservation relation for the solid in the following forms:

$$\frac{\partial}{\partial t}[\varepsilon\rho_f h_f + (1-\varepsilon)\rho_s h_s] + \frac{\partial}{\partial z}[u\rho_f h_f] = 0 \quad (1)$$

$$\frac{\partial(\varepsilon\rho_f)}{\partial t} + \frac{\partial}{\partial z}(u\rho_f) = 0. \quad (2)$$

This equation implies that no condensation, adsorption or phase change exist, so that the only mechanisms for fluid accumulation in the bed are the density change of the gas and/or the porosity change of the packing.

$$\frac{\partial}{\partial t}[(1-\varepsilon)\rho_s] = 0 \quad \text{or} \quad \rho_s(1-\varepsilon) = \text{constant}. \quad (3)$$

The latter equation indicates that a decrease in density due to swelling of the particles results in a decrease of the inter-particle void fraction, and that no expansion of the bed as a whole occurs. Equation (3) may be used in the following form to eliminate variations in porosity terms in equations (1) and (2):

$$\frac{\partial \varepsilon}{\partial t} = \frac{1-\varepsilon}{\rho_s}\frac{\partial \rho_s}{\partial t}. \quad (4)$$

Substitution of equation (4) in the developed form of equation (2) gives:

$$-\frac{\partial}{\partial z}(u\rho_f) = \varepsilon\frac{\partial \rho_f}{\partial t} + \frac{\rho_f}{\rho_s}(1-\varepsilon)\frac{\partial \rho_s}{\partial t}. \quad (5)$$

NOMENCLATURE

C_p	heat capacity [kJ kg^{-1} °C^{-1}]
D	dispersion coefficient [m^2 s^{-1}]
d_p	particle diameter [m]
$\mathscr{D}_m$	molecular diffusivity (equal to 0.18×10^{-4} for air) [m^2 s^{-1}]
G	mass flow rate per unit cross-sectional porous area [kg s^{-1} m^{-2}]
h	enthalpy [kJ kg^{-1}]
L	total bed length [m]
Pe	Péclet number based on column length due to hydrodynamic dispersion
Pe_{eq}	equivalent Péclet number due to variation of physical properties with temperature
R	heat capacity ratio, equation (7)
Re	Reynolds number, $(\rho_f u\, d_p/\mu_f)$
Sc	Schmidt number, $(\mu_f/\rho_f \mathscr{D}_m)$
T	temperature [°C]
t	time [s]
t_r	mean exit time of a temperature wave [s]
u	superficial velocity of fluid [m s^{-1}]
u_i	interstitial velocity of fluid [m s^{-1}]
z	space variable along axis of the column [m]
v_T	velocity of propagation [m s^{-1}]

Greek symbols

α	linear expansion coefficient [°C^{-1}]
β	cubical thermal expansion coefficient [°C^{-1}]
ε	void fraction of packed bed
μ	viscosity [Pl]
ρ	density [kg m^{-3}]
σ^2	variance of the temperature distribution

Subscripts

e	final
f	fluid
i	initial
s	solid
0	relative to the inlet of the column ($z = 0$).

Substitution of both equations (4) and (5) into (1) yields

$$\varepsilon\rho_f \frac{\partial h_f}{\partial t} + (1-\varepsilon)\rho_s \frac{\partial h_s}{\partial t} + u\rho_f \frac{\partial h_f}{\partial z} = 0. \qquad (6)$$

This form is identical to that which would be obtained by assuming that ε, ρ_f, ρ_s and u are constant. Here, all these quantities are functions of temperature. The enthalpy balance, equation (6), may be further developed by expressing the enthalpies in terms of specific capacities at constant pressure. Letting:

$$R = \frac{(1-\varepsilon)\rho_s C_{ps}}{\varepsilon \rho_f C_{pf}} \qquad (7)$$

be the heat capacity ratio (dimensionless), we can write equation (6) in the form:

$$\frac{\partial T_f}{\partial t} + R\frac{\partial T_s}{\partial t} + \frac{u}{\varepsilon}\frac{\partial T_f}{\partial z} = 0 \qquad (8)$$

with the assumption of thermal equilibrium, $T = T_f = T_s$, and defining the interstitial velocity as:

$$u_i = u/\varepsilon \qquad (9)$$

equation (8) becomes:

$$\frac{\partial T}{\partial t} + \frac{u_i}{1+R}\frac{\partial T}{\partial z} = 0. \qquad (10)$$

This quasi-linear, homogeneous, first-order partial differential equation is a 'kinematic wave' equation, and expresses the propagation of a value of temperature T at a velocity v_T such that:

$$v_T = \left(\frac{\partial z}{\partial t}\right)_T = -\frac{\partial T/\partial t}{\partial T/\partial z} = \frac{u_i}{1+R}. \qquad (11)$$

It is possible to associate uniquely a velocity to any temperature if we can express u_i and R as a function of temperature alone. This is a relatively simple matter for R, which contains only physical properties, that are supposed to be known as a function of T: ρ_s, ρ_f, C_{pf}, C_{ps}. The porosity ε is related to ρ_s through equation (3) and therefore known as a function of T. On the other hand, in order to express the velocity u, we need to use the fluid conservation equation (2), together with the dependence on T of the physical parameters.

This is done as follows. Let us introduce the thermal expansion coefficients of both phases:

$$\beta = -\frac{1}{\rho}\frac{\partial \rho}{\partial T}. \qquad (12)$$

Using equation (4), the mass balance on the fluid, equation (2) may then be rewritten as:

$$-\left(\beta_f + \frac{1-\varepsilon}{\varepsilon}\beta_s\right)\frac{\partial T}{\partial t} - u_i\beta_f\frac{\partial T}{\partial z} + \frac{\partial u_i}{\partial z} = 0. \qquad (13)$$

$\partial T/\partial t$ may be eliminated from equation (13) by using (10) to get:

$$f(T)\frac{\partial T}{\partial z} + \frac{1}{u_i}\frac{\partial u_i}{\partial z} = 0 \qquad (14)$$

with

$$f(T) = \frac{1}{1+R}\left(\frac{1-\varepsilon}{\varepsilon}\beta_s - R\beta_f\right). \qquad (15)$$

The velocity u_i may then in principle be obtained as a function of temperature by integration of equation (14) over the variable z, considering $f(T)$ as a known function of T—this yields:

$$\ln u_i(z, t) = \ln u_{i0}(t) + \int_{T_0}^{T(z,t)} f(T)\,\mathrm{d}T \qquad (16)$$

where $u_{i0}(t)$ is the value of the interstitial velocity at the bed inlet ($z = 0$), which may be a function of time, depending on the inlet condition. T_0 is the inlet temperature. Equation (16) may be written, after taking the exponential

$$u_i = u_{i0}(t)g(T_0, T) \qquad (17)$$

where $g(T_0, T)$ is a supposedly known function of the inlet temperature and of the local temperature.

The abscissa z of a temperature T at any time t may then be expressed by combining equations (11) and (17) and integrating over time:

$$z(t) - z(t_0) = \int_{t_0}^{t} v_T\,\mathrm{d}t = \frac{g(T_0, T)}{1+R(T)}\int_{t_0}^{t} u_{i0}(t). \qquad (18)$$

However complex this procedure may seem in the general case, it will be seen to be quite straightforward in some common practical situations. In the following, we shall apply this approach to different fluid–solid systems, illustrating different thermal behaviours of the packed bed. The systems considered involve common glass or lead beads as solid; and air, carbon dioxide or water as fluid.

GAS–SOLID SYSTEMS

Simplified form of general solutions

The general equation can be considerably simplified when the fluid is a gas, as is shown below. For the air–glass system, one can see in Table A1 (Appendix) that in the temperature range 0–100°C, the relative variation of ρ_s is about 0.2%, more than 100 times smaller than the relative variation of ρ_f. This conclusion remains valid not only for the other gas–solid systems of Table A1 but also for other solids such as SiO_2, Al_2O_3 and other gases such as CH_4. The result is that the porosity variation due to dilatation may be neglected and that the term $(1-\varepsilon)\beta_s/\varepsilon$ in the function $f(T)$ [equation (15)] is negligible.

The equations of state of most gases under usual conditions imply:

$$\rho_f T = \text{constant} \qquad (19)$$

(as is the case for example for the ideal gas law or Van der Waals' equation). The thermal expansion coefficient is then:

$$\beta_f = -\frac{1}{\rho_f}\frac{\partial \rho_f}{\partial T} = \frac{1}{T}. \qquad (20)$$

Moreover, owing to the large value of R (between 1000 and 4000, see Table 1), the ratio $R/(1+R)$ is practically constant and equal to 1.

With these approximations, equation (14) becomes simply:

$$\frac{1}{T}\left(\frac{\partial T}{\partial z}\right)_t = \frac{1}{u_i}\left(\frac{\partial u_i}{\partial z}\right)_t \qquad (21)$$

which is integrated into:

$$\frac{u_i(z, t)}{u_i(0, t)} = \frac{T(z, t)}{T(0, t)} \quad \text{or} \quad \frac{u}{T} = \text{constant}. \qquad (22)$$

The local velocity is thus simply related to the local temperature and to the inlet conditions ($z = 0$). Notice also that the specific mass flux G (mass flow rate per unit cross-sectional porous area) is the same at any axial position and equal to its inlet value. This results from equations (19) and (22), which allows us to write:

$$G = \rho_f u_i = \rho_{f0} u_{i0} = G_0 \qquad (23)$$

the subscript 0 referring to the bed inlet ($z = 0$). Equations (19), (22) and (23) imply that any change in mass flow rate and/or temperature at the bed inlet will immediately affect the velocity of all temperatures downstream.

Table 1. Gas–solid systems. Values of R [equation (7)] and $\rho_f(1+R)$ (physical properties from Table A1)

T	Air–glass		Air–lead		CO_2–glass		CO_2–lead	
(°C)	R	$\rho_f(1+R)$	R	$\rho_f(1+R)$	R	$\rho_f(1+R)$	R	$\rho_f(1+R)$
0	2188	2830	1687	2183	1753	3445	1352	2657
10	2314	2891	1750	2187	1836	3481	1388	2632
20	2442	2946	1815	2190	1920	3515	1427	2613
30	2577	3006	1881	2194	2004	3549	1462	2590
40	2713	3064	1945	2197	2092	3585	1500	2571
50	2855	3124	2011	2201	2181	3622	1536	2551
60	2994	3178	2076	2204	2268	3653	1573	2534
70	3142	3237	2143	2208	2359	3688	1608	2515
80	3290	3294	2207	2210	2452	3726	1645	2500
90	3443	3354	2273	2215	2545	3760	1680	2483
100	3593	3407	2338	2217	2636	3793	1716	2467

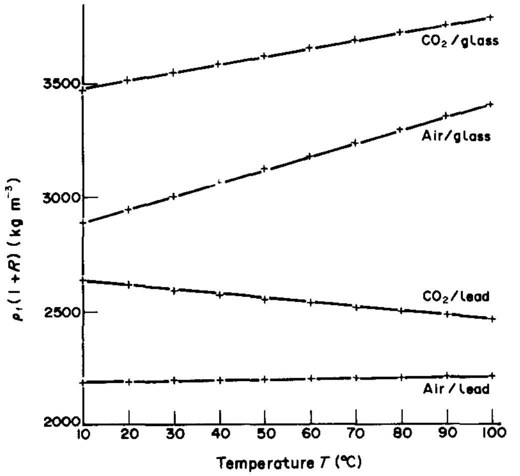

FIG. 1. Variation with temperature of the product $\rho_f(1+R)$ for different gas–solid pairs ($p = 1$ atm).

The instantaneous velocity of propagation v_T of any temperature, given by equation (11) may now be expressed as:

$$v_T = \frac{u_i}{1+R} = \frac{u_{i0}T}{(1+R)T_0} = \frac{G_0}{\rho_f(1+R)}. \qquad (24)$$

The denominator is a function only of the temperature considered; G_0 may be a function of time. If G_0 is constant, v_T is a constant for a given temperature. The abscissa of the temperature T at time t is given in the general case by equation (18) as:

$$z(t) - z(t_0) = \frac{1}{\rho_f(1+R)} \int_{t_0}^{t} G_0 \, \mathrm{d}t. \qquad (25)$$

The case of sharp temperature steps. Dispersive and compressive behaviour

Let us now consider the effect of a sharp change in inlet temperature (step change) at time $t = 0$, while the mass flux G_0 is maintained constant. This step disturbance generates a moving temperature distribution in the bed (a wave) from the initial to the final value of T. Each value of T in this distribution moves at any time with a constant velocity v_T given by equation (24). Since $\rho_f(1+R)$ depends on temperature, each value of T moves in general with a different velocity, and therefore the shape of the wave is expected to change as it propagates.

To illustrate this, let us consider first the air–glass system, and submit the packed bed to a heating process, that is to a positive temperature step. For example, starting with the bed uniformly at 20°C, we increase the inlet air temperature from 20 to 100°C. The data of Table 1, represented in Fig. 1, show that in this temperature range, $\rho_f(1+R)$ increases with temperature, practically linearly and by about 17%. Equation (24) thus implies that high temperatures move slower than low temperatures, and therefore the temperature wave tends to spread as it propagates through the bed, as illustrated by Fig. 2.

This so-called 'dispersive' *behaviour is thus the result of variations in densities and heat capacities (capacitive factors), and not of flow dispersion (convective factor), axial diffusion (diffusive factors), or finite rates of heat transfer (resistive factors), which have been neglected in the present model.* In physical reality, the four types of factors mentioned will contribute to the overall dispersion. Notice also that the linearity of $\rho_f(1+R)$ with T entails that the temperature distributions T vs z resulting from a perfectly sharp step are straight (Fig. 2).

Applying the same approach to the other systems, it is seen that a similar dispersive behaviour is obtained with the CO_2–glass system, although the corresponding wave is less dispersed, because the variation of $\rho_f(1+R)$ is smaller (less than 10%). In the system air–lead, the product $\rho_f(1+R)$ is practically constant, and therefore there is no dispersion due to the capacitive factors.

By contrast, in the CO_2–lead system, $\rho_f(1+R)$ decreases as temperature increases, and thus high temperatures tend to move faster than low temperatures. If the inlet disturbance is not a sharp jump but say a ramp, the resulting wave is initially spread, but will tend to sharpen or 'recompress'. This so-called '*com-*

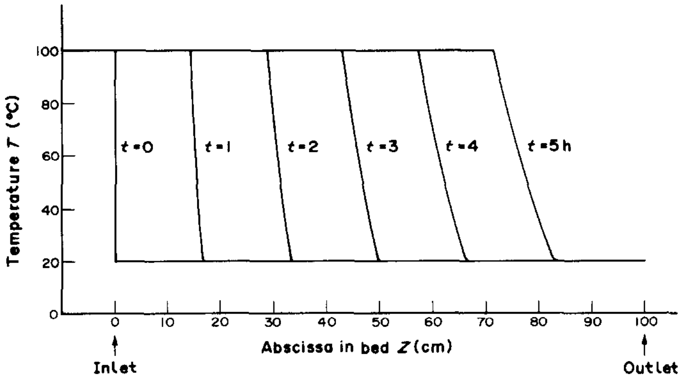

FIG. 2. Propagation of a temperature wave in a packed bed of glass beads percolated by air, illustrating dispersive behaviour of an initially sharp wave (calulated). $G_0 = 488$ kg m^{-2} h^{-1}; initial $T = 20$°C; inlet $T = 100$°C.

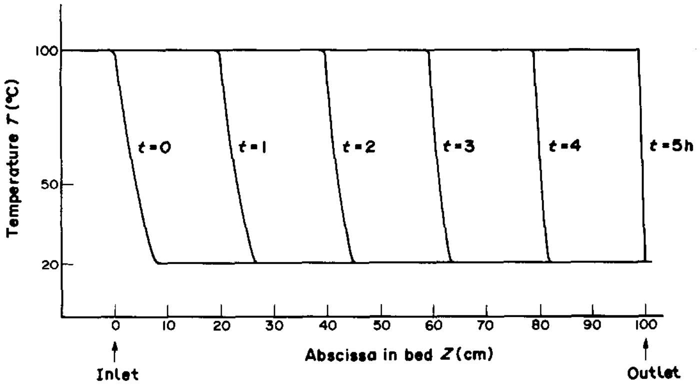

FIG. 3. Propagation of a temperature wave in a packed bed of lead particles percolated by CO_2, illustrating compressive behaviour of an initially dispersed wave (calculated). $G_0 = 488$ kg m^{-2} h^{-1}; initial $T = 20°C$; inlet $T = 100°C$.

pressive' behaviour is illustrated in Fig. 3. Clearly, if the process was continued, say in a longer column, or alternately if the inlet disturbance were sharp, a perfectly sharp wave (a discontinuity in temperature) would result. From then on, the higher temperatures can no longer move faster than the lower ones, since this would result in an 'overhanging', multivalued temperature distribution—a physically unrealistic situation. The physical reality is the formation of a *steady shock*, a non-equilibrium situation in which the sharpening tendency due to the capacity factors is counterbalanced by the dispersive effects of convective, diffusive and resistive factors. The wave then takes a steady shape, a constant pattern, such that all values of temperatures move at the same velocity. Thermal equilibrium between the two phases may no longer be assumed, and the detailed description of the shape of the shock wave requires accounting for a finite rate of heat transfer. The formation of shocks as the result of a compressive behaviour is conceptually and mathematically the same as that encountered in aerodynamics, hydraulics, chromatography [11, 12], heat storage with phase change [13, 14], and is studied at length in the literature of these fields. Our purpose here is merely to show that such behaviour may occur in common systems under the effect of the dependency of density and heat capacity on temperature.

It is straightforward to transpose the foregoing discussion to the case of a cooling process, that is to a decrease in inlet gas temperature. The type of behaviour observed is then reversed: the air–glass system will display a compressive behaviour and the CO_2–lead system a dispersive behaviour. A reversal of the behaviour is also obtained by flow reversal; for example, if the heating process of glass beads with air is stopped while the dispersive temperature wave is still in the bed, and cooling is started by flowing cold air at the previously outlet end (thus by reversing the flow), the temperature wave recompresses and eventually tends to become a shock [14].

A more quantitative evaluation of these effects is presented in a later section.

LIQUID–SOLID SYSTEMS

In liquid–solid systems, it may appear reasonable to neglect the thermal expansion of both phases, and to account only for variations of specific heats, that is of R. The useful consequence of this assumption is that the velocity u_i may be considered as constant. We shall first assess the validity of this assumption by comparing the results of the rigorous and the approximate calculations.

Let us consider the water–glass system, for which the pertinent physical data are reported in Table A1. The function $f(T)$ [equation (15)] is calculated vs temperature (Table 2) and plotted on Fig. 4, together

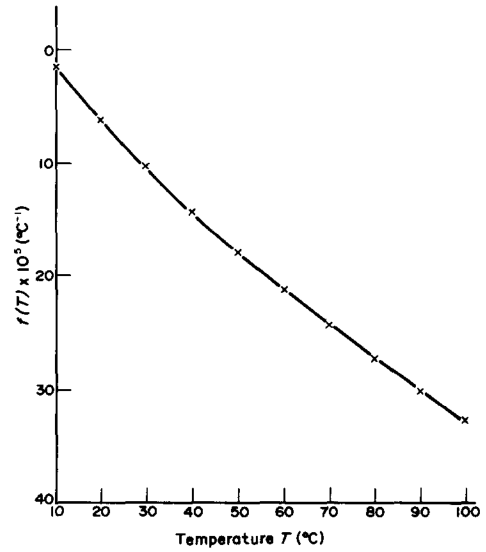

FIG. 4. Variation with temperature of the function f [equation (15)]. × × × × Data points from Table 2; —— polynomial fitting.

Table 2. Water–glass system

T (°C)	R	ε	$R\beta_f$ $\times 10^5$	$(1-\varepsilon)\beta_s/\varepsilon$ $\times 10^5$	$f(T)$ $\times 10^5$	$f^*(T)$ $\times 10^5$
10	0.6894	0.39986	5.66	3.8272	−1.085	−1.61
20	0.7068	0.39969	14.60	3.8300	−6.310	−6.05
30	0.7249	0.39954	22.15	3.8323	−10.621	−10.24
40	0.7428	0.39940	28.90	3.8346	−14.529	−14.17
50	0.7614	0.39923	35.21	3.8373	−17.972	−17.84
60	0.7795	0.39909	41.22	3.8395	−21.006	−21.26
70	0.7989	0.39892	47.13	3.8423	−24.065	−24.42
80	0.8184	0.39877	52.98	3.8447	−27.020	−27.33
90	0.8382	0.39863	58.84	3.8469	−29.914	−29.98
100	0.8573	0.39846	64.70	3.8496	−32.764	−32.37

$*f^*(T) = 1.28\times 10^{-8}T^2 - 4.826\times 10^{-6}T + 3.08667\times 10^{-5}$.

with an analytical fitting by a third-degree polynomial:

$$f^*(T) = aT^2 + bT + c \quad (T \text{ in } ^\circ\text{C}) \tag{26}$$

where

$$a = 1.280\times 10^{-8};\quad b = -4.826\times 10^{-6};$$
$$c = 3.087\times 10^{-5}.$$

Equation (14) is now rewritten:

$$(aT^2+bT+c)\frac{\partial T}{\partial z} + \frac{1}{u_i}\frac{\partial u_i}{\partial z} = 0 \tag{27}$$

which may be integrated on z to give:

$$u_i = u_{i0}\exp\left[P(T_0) - P(T)\right] \tag{28}$$

with

$$P(T) = \int f^*(T)\,\mathrm{d}T = \frac{aT^3}{3} + \frac{bT^2}{2} + cT$$

u_{i0} and T_0 are values at the bed inlet ($z = 0$). For a step change of the inlet temperature at $z = 0$, and with constant inlet flow rate u_{i0}, equations (11) and (18) become:

$$v_T = u_{i0}\frac{\exp\left[P(T_0) - P(T)\right]}{1 + R(T)} \tag{29}$$

$$z = v_T t \tag{30}$$

which have been used to calculate the values of v_{T_1} in Table 3 and to generate the profiles of Fig. 5. v_T is seen to decrease as temperature increases (high temperatures are slower than low temperatures), and therefore the heating wave of Fig. 5 disperses as it propagates in the bed.

If the thermal expansion of both water and glass are neglected [β_f and β_s are zero in equation (15)], $f(T)$ is zero and the velocity v_T [equation (29)] is given more simply by:

$$v_T = \frac{u_{i0}}{1+R} \tag{31}$$

The values v_{T_2} calculated using this equation are reported in Table 3 and compared to the rigorous values v_{T_1}. The differences are less than 2%, and certainly less than the uncertainty of the values of the physical properties. It is therefore clear that for the water–glass system, and in the temperature range considered, neglecting thermal expansion introduces no significant error.

QUANTITATIVE EVALUATION OF THE DISPERSIVE EFFECT

The purpose of the present section is to compare the quantitative importance of the wave spreading due

Table 3. Water–glass system: Propagation velocities. Positive step change: Inlet temperature $T(z = 0) = 100$°C. Initial temperature uniform in the bed: 20°C, $u_{i0} = 22.54$ m h^{-1}

T (°C)	$P(T)$	v_{T_1}(m h^{-1}) × 10 equation (29)	v_{T_2}(m h^{-1}) × 10 equation (31)	$(v_{T_1} - v_{T_2})/v_{T_1}$ (%)
20	3.14×10^{-4}	1.296	1.319	−1.7
30	1.13×10^{-3}	1.283	1.305	−1.6
40	2.35×10^{-3}	1.274	1.292	−1.5
50	3.96×10^{-3}	1.260	1.275	−1.3
60	5.91×10^{-3}	1.251	1.265	−1.1
70	8.20×10^{-3}	1.242	1.251	−0.9
80	1.079×10^{-2}	1.229	1.238	−0.6
90	1.366×10^{-2}	1.220	1.224	−0.3
100	1.678×10^{-2}	1.211	1.211	0.0

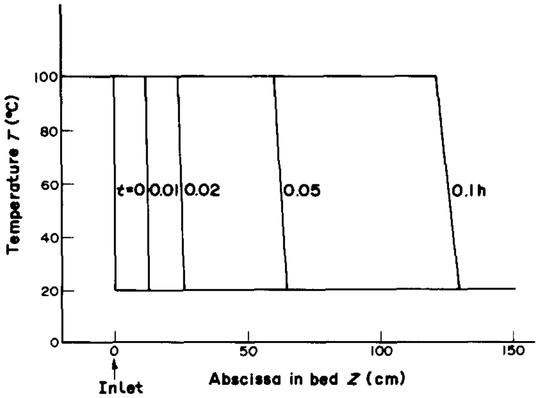

FIG. 5. Propagation of a dispersive temperature wave in a packed bed of glass beads percolated by hot water (calculated). $u_{i0} = 22.5\,\mathrm{m\,h^{-1}}$; initial $T = 20°\mathrm{C}$; inlet $T = 100°\mathrm{C}$.

to variations in physical properties to the spreading due to common convective axial dispersion. For this purpose, let us consider the dispersive heating waves for the air–glass and the water–glass systems (Figs. 2 and 5), and let us treat these waves as if their spreading resulted only from convective axial dispersion of the fluid. Using the properties of the classical model for axially dispersed plug flow in packed beds [1, 2], we can relate the measured variance σ^2 of the temperature distribution to an equivalent column Péclet number by:

$$Pe = \frac{uL}{D} = \frac{2}{\sigma^2}. \qquad (32)$$

The variance is measured, as indicated for example by Villermaux [2] and summarized in Fig. 6, on a normalized temperature vs reduced time curve. These curves have been drawn on Fig. 7 for the two systems considered from the profiles of Figs. 2 and 5, with a bed length of 1 m. The equivalent column Péclet numbers obtained via equation (32) are:

$$\text{air–glass} \quad Pe_{\mathrm{eq}} \sim 600$$

$$\text{water–glass} \quad Pe_{\mathrm{eq}} \sim 2700.$$

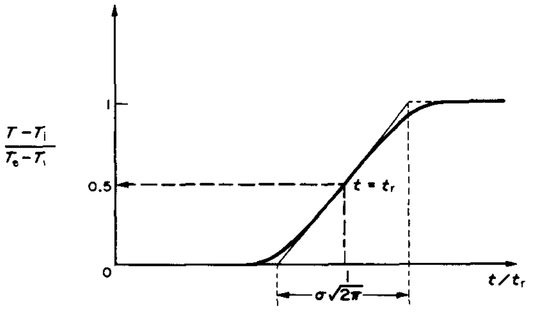

FIG. 6. Graphical determination of standard deviation σ on normalized effluent temperature history (schematic) (t_r = mean exit time of the curve).

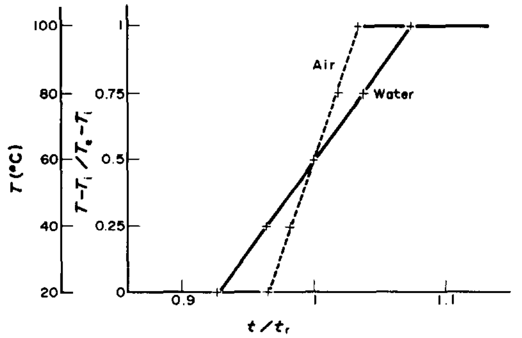

FIG. 7. Normalized temperature histories in a bed of glass beads percolated by air (– – – –) and by water (——) (calculated). t_r(air) = 6.51 h; t_r(water) = 0.08 h.

The column length, the particle diameter and the mass flow rates in the two systems were chosen so that the flow regimes be the same. More precisely, we took:

$$ReSc(\text{air}) = Re(\text{water})$$

with values of 0.1, 1 and 10, for particles diameters of 0.04, 0.4 and 4 mm; G and L were kept constant at the values of Figs. 2 and 5.

The hydrodynamic particle Péclet numbers under these conditions can be estimated from the published correlations with the Reynolds number or the product $ReSc$(for gases) [2, 3] and the column Pe is then simply obtained by multiplying by the ratio L/d_p of bed to particle size. We then get the results listed in Table 4.

Comparing Pe to the values of Pe_{eq}, it follows that:

—the equivalent Péclet numbers are equal to or much smaller than Pe in the air–glass system, which means that the spreading of the temperature wave due to variation of the gas density and of the ratio of specific heats is of the same order or much greater than the spreading due to convective dispersion, and may not be neglected without care;

—in the water–glass system, Pe_{eq} is about 20 times larger than Pe for $d_p = 4$ mm, but five times smaller for $d_p = 0.04$ mm, therefore the spreading due to variation of specific heats may be safely neglected with respect to convective dispersion in the first case but not in the second case.

The equivalent Péclet numbers are found to be about 90,000 for the air–lead system, implying in general a negligible contribution to spreading, and about 2200 for the CO_2–glass system, implying a negligible spreading only at high values of $ReSc$.

Table 4

Gas–solid		Liquid–solid	
ReSc	*Pe* (column)	*Re*	*Pe* (column)
0.1	10,000	0.1	12,500
1	5750	1	1250
10	625	10	125

CONCLUSIONS

The dependence on temperature of the densities and specific heats of the two phases (capacitive effect) may in some cases affect significantly the shape and deformation of the temperature waves generated in percolated packed beds by changes in the fluid temperature. Depending on the properties of the two phases and on the direction of the temperature change, the effect may be either to spread the wave more and more as it propagates, or on the contrary, to sharpen it to a constant-shaped shock.

This effect is likely to be negligible with respect to convective axial dispersion in liquid–solid systems, at least for particles larger than 1 mm. The effect is likely to be larger and to over-run convective dispersion in gas–solid systems and also for smaller particles (convective dispersion is then smaller and fluid–solid transfer faster, while the capacity effect is not affected by particle size). However, no general conclusion should be drawn, and each particular case should be carefully examined. This may be especially important when temperature waves are used for example to obtain fluid–solid heat transfer coefficients. These may be underestimated when the dispersive behaviour is neglected; the spreading measured may then be attributed unduly to a low transfer coefficient.

On the contrary, when a compressive effect exists, the sharpness of the wave may be attributed unduly to a high heat transfer coefficient, which will thus be overestimated.

The approaches presented here are suitable for evaluating the quantitative importance of the capacitive effect. For this purpose, in liquid–solid systems, the thermal expansion of the phases may be neglected, and the linear interstitial velocity may be considered independent of temperature. Only the variation in specific heats (the ratio R) needs to be accounted for. In gas–solid systems, the approach implies negligible pressure drop in the packing, with respect to the absolute pressure. Thermal expansion of the solid may be neglected, and it may be assumed that the gas density is inversely proportional to temperature. Then the local interstitial velocity is proportional to the local temperature and the mass flux is the same throughout the bed (of course, it is assumed no adsorption or phase change occurs). The velocity distribution, thus the wave spreading, is then determined by the variation with T of the product $\rho_f(1+R)$.

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APPENDIX: PHYSICAL PROPERTIES OF SOLIDS AND FLUIDS

The materials considered previously are common glass and lead as solids; air, carbon dioxide and water as fluids. The variation of densities and heat capacities with temperature are indicated in Table A1. In the calculations, algebraic expressions for the solids and the gases were used, which are given below. For water, the data from (15) and (19) were used in numerical form.

For solids [15–17]

$$\rho_s = \frac{\rho_s(0)}{1+\beta T} \quad (\rho_s \text{ in kg m}^{-3})$$

Table A1. Variation of physical properties with temperature

	Glass		Lead		Air		CO_2		Water		
T (°C)	ρ_s (kg m^{-3})	C_{p_s} (kJ kg^{-1} °C^{-1})	ρ_s (kg m^{-3})	C_{p_s} (kJ kg^{-1} °C^{-1})	ρ_f (kg m^{-3})	C_{p_f} (kJ kg^{-1} °C^{-1})	ρ_f (kg m^{-3})	C_{p_f} (kJ kg^{-1} °C^{-1})	$\rho_f \times 10^{-3}$ (kg m^{-3})	C_{pf} (kJ kg^{-1} °C^{-1})	$\beta_f \times 10^{-3}$ (K^{-1})
0	2500	0.753	11,374.2	0.1276	1.293	0.998	1.964	0.820	0.9999	4.216	
10	2499.4	0.770	11,364.3	0.1280	1.249	0.999	1.895	0.830	0.9997	4.191	0.0821
20	2498.7	0.786	11,354.4	0.1284	1.206	1.001	1.830	0.839	0.9982	4.181	0.2066
30	2498.1	0.803	11,344.6	0.1288	1.166	1.002	1.770	0.849	0.9957	4.177	0.3056
40	2497.5	0.820	11,334.8	0.1292	1.129	1.004	1.713	0.858	0.9923	4.178	0.3890
50	2496.8	0.837	11,324.9	0.1296	1.094	1.005	1.660	0.867	0.9881	4.180	0.4624
60	2496.2	0.853	11,315.1	0.130	1.061	1.007	1.610	0.876	0.9832	4.183	0.5288
70	2495.5	0.870	11,305.4	0.1304	1.030	1.008	1.563	0.885	0.9778	4.188	0.5900
80	2494.9	0.887	11,295.6	0.1308	1.001	1.010	1.519	0.893	0.9718	4.195	0.6473
90	2494.3	0.904	11,285.8	0.1312	0.974	1.011	1.477	0.902	0.9653	4.204	0.7019
100	2493.6	0.920	11,276.1	0.1316	0.948	1.013	1.438	0.910	0.9584	4.215	0.7547

with β the cubical expansion coefficient: $\beta = 3\alpha$; and α being the linear expansion coefficient

$$C_{p_s} = a + bT \quad (C_{p_s} \text{ in kJ kg}^{-1}\,{}^\circ\text{C}^{-1})$$

with

	$\rho_s(0)$ (kg m^{-3})	α (°C^{-1})	a (kJ kg^{-1} °C^{-1})	b (kJ kg^{-1} °C^{-2})
Common glass	2500	85×10^{-7}	0.7535	1.67×10^{-3}
Lead	11,374	29×10^{-6}	0.1165	4.078×10^{-5}

For gases [18]

$$\rho_f = \frac{PM}{RT} = \frac{\text{cste}}{T} \quad (\rho_f \text{ in kg m}^{-3}, T \text{ in K})$$

$$C_{p_f} = (a + bT + cT^2 + dT^3) \times 4.183/M \quad (C_{p_f} \text{ in kJ kg}^{-1}\,\text{K}^{-1}, T \text{ in K})$$

with

$P = 1$ atm

$R = 0.0821$ atm m^3 kmol^{-1} K^{-1}

Gas	M (kg kmol^{-1})	a	b	c	d
Air	29	6.713	4.697×10^{-4}	11.47×10^{-7}	-4.696×10^{-10}
CO_2	44	5.316	142.85×10^{-4}	-83.62×10^{-7}	17.84×10^{-10}

TRANSFERT THERMIQUE DANS UN LIT FIXE PERCOLATEUR AVEC EFFETS DES PROPRIETES PHYSIQUES VARIABLES AVEC LA TEMPERATURE

Résumé—Cette étude concerne les ondes de température créées dans un lit fixe par un changement de température du fluide percolant. La forme et la déformation de ces ondes dépend considérablement de la variation des propriétés physiques avec la température (chaleurs spécifiques et masses spécifiques). On illustre théoriquement deux comportements qualitativement différents. Par exemple si de l'air, traversant un lit de billes de verre, subit un échelon de chauffe à l'entrée (de 20 à 100°C), l'onde de température qui en résulte est 'dispersive', en ce sens qu'elle s'élargit de plus en plus à mesure qu'elle se propage, sous le seul effet de variation des propriétés physiques des deux phases. Le même comportement est observé, avec des intensités plus faibles, pour les systèmes CO_2–verre et eau–verre. Au contraire, un lit de grains de plomb percolé par du gaz carbonique donne lieu à un comportement 'compressif', qui engendre une 'onde de choc' de forme abrupte et constante. Ces effets sont étudiés à l'aide d'un modèle simple qui néglige la dispersion convective et l'écart à l'équilibre thermique local des deux phases, pour ne prendre en compte que la dépendance des chaleurs et masses spécifiques à l'égard de la température. L'importance quantitative de l'effet dispersif est comparé à la dispersion convective, en évaluant les nombres de Péclet respectifs. Suivant les conditions opératoires et le système considéré, l'effet étudié peut être prédominant (cas fréquent des systèmes gaz–solide, et/ou des nombres de Reynolds faibles) ou négligeable (cas fréquent des systèmes liquide–solide, et/ou des nombres de Reynolds élevés). Il peut être important de tenir compte de ces effets pour interpréter des expériences de transfert de chaleur en lit fixe, en particulier lorsqu'il s'agit d'estimer des coefficients de transfert à partir de telles expériences.

DER WÄRMEÜBERGANG IN DURCHSTRÖMTEN FESTBETTEN MIT TEMPERATURABHÄNGIGEN PHYSIKALISCHEN EIGENSCHAFTEN— EINSCHNÜR- UND AUSBREITUNGSEFFEKTE

Zusammenfassung—Diese Untersuchung bezieht sich auf Temperaturwellen, die in einem Festbett durch Änderung der Eintrittstemperatur des strömenden Fluids erzeugt werden. Die Form dieser Wellen und ihre Verformung während des Fortschreitens können durch die temperaturbedingte Änderung der spezifischen Wärmen und der spezifischen Volumina stark beeinflußt werden. Zwei qualitativ unterschiedliche Verhaltensweisen werden theoretisch dargelegt. Wird beispielsweise die Temperatur der durch ein Bett aus Glaskugeln strömenden Luft von 20°C auf 100°C erhöht, so ist die resultierende Temperaturwelle in dem Sinne dispers, daß sie sich unter dem Einfluß der Änderungen von Dichte und Wärmekapazität der beiden Phasen, beim Fortschreiten mehr und mehr verbreitert. Ein ähnlicher aber schwächerer Effekt tritt für die Systeme CO_2/Glas und Wasser/Glas auf. Andererseits wird ein Bett aus Bleikugeln, das von Kohlendioxid durchströmt wird, eine einengende Wirkung aufweisen, die in einer stetigen und steilen 'Schock-Welle' zum Ausdruck kommt. Alle Tendenzen kehren sich um, wenn statt einer Temperaturerhöhung eine

Temperaturerniedrigung aufgeprägt wird. Die genannten Effekte werden anhand eines einfachen Modells, das Wärmeausbreitung durch Konvektion und Wärmeübergangswiderstände zwischen den beiden Phasen vernachlässigt, dargelegt. Die quantitative Bedeutung der Spreizung wird mit der aufgrund von konvektiver axialer Ausbreitung verglichen, wofür die korrespondierenden Péclet-Zahlen ermittelt werden. Abhängig von den Betriebsbedingungen können die dargelegten Einflüsse bestimmend sein (meist in Gas/Feststoff-Systemen und/oder bei kleinen Reynolds-Zahlen) oder vernachlässigbar (meist in Flüssig/Feststoff-Systemen und/oder bei hohen Reynolds-Zahlen). Die Berücksichtigung des Ausbreitungseffekts (oder Einengungseffekts) durch die Änderung der Eigenschaften scheint bei der Interpretation von Festbettexperimenten wichtig zu sein, wenn beispielsweise Wärmeübergangskoeffizienten aus derartigen Experimenten bestimmt werden.

ТЕПЛОПЕРЕНОС В ФИЛЬТРУЮЩИХ НЕПОДВИЖНЫХ СЛОЯХ С ЗАВИСЯЩИМИ ОТ ТЕМПЕРАТУРЫ ФИЗИЧЕСКИМИ СВОЙСТВАМИ. ЭФФЕКТЫ СЖАТИЯ И РАСШИРЕНИЯ

Аннотация—Изучаются температурные волны, возникающие в плотном слое при изменении входной температуры фильтрующейся жидкости. Форма волн и их деформация при распространении могут существенно зависеть от изменения удельной теплоемкости и объема с температурой. Теоретически рассмотрены два качественно различающихся режима. Например, в случае возрастания температуры воздуха, проходящего через слой стеклянных шариков, от 20 до 100°С результирующая температурная волна является “расширяющейся” в том смысле, что при распространении под действием изменения плотностей и теплоемкостей обеих фаз она все больше и больше расширяется. Подобный, но более слабый, эффект имеет место для систем CO_2–стеклянные шарики и вода–стеклянные шарики. Наоборот, в случае фильтрации двуокиси углерода через слой свинцовых шариков проявляется “сжимающий” эффект, приводящий к устойчивой и резкой “ударной волне”. Тенденция обратна, когда вместо нагрева рассматривается охлаждение. Обнаруженные эффекты сжатия и расширения проиллюстрированы с помощью упрощенной модели без учета конвективной дисперсии и теплового сопротивления между фазами. Качественное влияние расширения сравнивается путем оценки соответствующих чисел Пекле с вкладом, вносимым конвективной осевой дисперсией. В зависимости от рабочих условий эффект может быть преобладающим (в основном в системах газтвердое тело и/или при низких значениях чисел Рейнольдса) или пренебрежимо малым (в основном в системах жидкость–твердое тело и/или при больших числах Рейнольдса). Учет эффекта расширения (или сжатия), вызванный изменением свойств, необходим при рассмотрении результатов экспериментов с неподвижным слоем, например, при оценке коэффициентов теплопереноса.